OCR
Oxford Cambridge and RSA

Tuesday 9 June 2015 – Morning

A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 The displacement, x m, of a particle at time t s is given by the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+8\frac{\mathrm{d}x}{\mathrm{d}t}+25x=0.$$

Initially the particle is at the origin and has a velocity of $\frac{1}{4}\,\mathrm{m\,s^{-1}}$.

(i) Find the particular solution for x. **[8]**

(ii) Find the maximum displacement of the particle from its initial position, giving your answer correct to 3 significant figures. **[4]**

(iii) Describe the behaviour of your solution for large values of t. **[1]**

In a different situation, an additional force is applied to the particle and the differential equation satisfied by x is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+8\frac{\mathrm{d}x}{\mathrm{d}t}+25x=5\sin 5t.$$

(iv) Using the same initial conditions as in part **(i)**, find the new particular solution for x. **[10]**

(v) Describe the behaviour of your new solution for large values of t. **[1]**

2 The differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}-ny=2x-1,$$

where n is a non-zero constant, is to be solved for $x>0$.

Firstly consider the case $n\neq 1$.

(i) Find the general solution for y in terms of x and n. **[8]**

(ii) For $n=-1$, find the equation of the solution curve that passes through the point $(2,0)$ and sketch the curve for $x>0$. **[4]**

Now consider the case $n=1$.

(iii) Find the general solution for y in terms of x. **[5]**

(iv) Show that the solution curve for which $y=0$ when $x=1$ has exactly one stationary point. **[3]**

Now consider the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x}-y=\frac{1}{\sqrt{2x-1}}.$$

(v) Use Euler's method, with a step length of 0.1 and initial conditions $y=0$ when $x=1$, to estimate y when $x=1.3$. The algorithm is given by $x_{r+1}=x_r+h$, $y_{r+1}=y_r+hy'_r$. **[4]**

3 The resistance to motion of a small test car of mass 20 kg is modelled differently according to the aerodynamic features of the bodywork being tested. The motion of the test car is studied as it moves in a horizontal straight line. In each trial, the car is initially at rest at A; at time t s its velocity is $v\,\mathrm{m\,s^{-1}}$ and its distance from A is x m. The only horizontal forces acting on the car are a driving force of 100 N and a varying resistance force of magnitude R N.

In the first trial, the resistance to motion is modelled by $R = 4v^2$.

(i) Write down and solve a differential equation to show that

$$v^2 = 25\left(1 - \mathrm{e}^{-\frac{2}{5}x}\right).$$

Find the value of v when $x = 10$. **[9]**

(ii) Find the value of t when $x = 10$. **[7]**

In the second trial, the resistance to motion is modelled by $R = 2v$.

(iii) Write down and solve a differential equation to find v in terms of t. State the terminal velocity of the car. **[7]**

(iv) Find the value of t in the second trial when the car's speed is equal to the value of v found in part **(i)**. **[1]**

4 Two species of small rodent, X and Y, compete for survival in the same environment. The populations of the species, at time t years, are x and y respectively and they are modelled by the simultaneous differential equations

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2(x - y),$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{3}{8}\left(x - 80\mathrm{e}^{-\frac{1}{2}t}\right).$$

(i) Show that

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 2\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{3}{4}x = 60\mathrm{e}^{-\frac{1}{2}t}.$$

Find the general solution for x. **[10]**

(ii) Find the corresponding general solution for y. **[3]**

When $t = 0$, $x = 40$ and $y = 50$.

(iii) Find the particular solutions for x and y. **[4]**

(iv) Find the time T at which the model predicts that the rodents of species X will die out. Find the population of species Y predicted at this time. **[6]**

(v) Comment on the suitability of the model for times greater than T. **[1]**

END OF QUESTION PAPER

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Candidate forename		Candidate surname	

Centre number						Candidate number				

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1 (i)

(answer space continued on next page)

1 (i)	**(continued)**
1 (ii)	

1 (iii)	
1 (iv)	
	(answer space continued on next page)

1 (iv)	**(continued)**
1 (v)	

2 (i)	

2 (ii)	

2 (iii)	
2 (iv)	
	(answer space continued on next page)

2 (iv)	**(continued)**
2 (v)	

3 (i)

3 (ii)	

3 (iii)	
3 (iv)	

4 (i)	

4 (ii)	
4 (iii)	
	(answer space continued on next page)

4 (iii)	**(continued)**
4 (iv)	
	(answer space continued on next page)

4 (iv)	**(continued)**
4 (v)	

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Mechanics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

When a value is given in the paper
Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

When a value is not given in the paper
Accept any answer that agrees with the correct value to 2 s.f.

ft should be used so that only one mark is lost for each distinct error made in the accuracy to which working is done or an answer given. Refer cases to your Team Leader where the same type of error (e.g. errors due to premature approximation leading to error) has been made in different questions or parts of questions.

There are some mistakes that might be repeated throughout a paper. If a candidate makes such a mistake, (eg uses a calculator in wrong angle mode) then you will need to check the candidate's script for repetitions of the mistake and consult your Team Leader about what penalty should be given.

There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	Guidance	
1	(i)		Auxiliary equation: $m^2+8m+25=0$	M1		
			$m=-4\pm 3i$	A1		
			CF: $x=e^{-4t}\left(A\cos 3t+B\sin 3t\right)$	F1	From their roots	
			$A=0$	M1	Use condition	
			$\dot{x}=e^{-4t}\left(3B\cos 4t\right)-4e^{-4t}.B\sin 3t$	M1	Differentiate, product rule	
			$0.25=3B:\ B=\frac{1}{12}$	M1 A1	Use condition (must use 0.25)	
			$x=\frac{1}{12}e^{-4t}\sin 3t$	F1		
				[8]		
	(ii)		$\frac{dx}{dt}=0$: $e^{-4t}\left(3\cos 3t-4\sin 3t\right)=0$	M1	Equate their $\dot{x}$ to zero	
			$\tan 3t=\frac{3}{4}:\ t=0.2145$	M1	Obtain expression for tan and attempt to solve	
				A1	Dependent on using correct answer to **(i)**	
			$x=0.0212$	A1	cao	
				[4]		
	(iii)		Oscillations with decreasing amplitude	B1	Accept amplitude $\to 0$; accept $x\to 0$. accept "oscillations with small amplitude" Do not accept "oscillations"	
				[1]		

Question			Answer	Marks	Guidance
	(iv)		CF: $x = e^{-4t}(C\cos 3t + D\sin 3t)$	F1	
			PI: $x = P\sin 5t + Q\cos 5t$	B1	Correct form
			$\dot{x} = 5P\cos 5t - 5Q\sin 5t$ $\ddot{x} = -25P\sin 5t - 25Q\cos 5t$	M1	Differentiate twice and substitute
			$-25P - 40Q + 25P = 5$ $-25Q + 40P + 25Q = 0$:	M1	Compare coefficients and solve
			$P = 0, Q = -\frac{1}{8}$	A1	
			$x = e^{-4t}(C\cos 3t + D\sin 3t) - \frac{1}{8}\cos 5t$	F1	Their CF with 2 arbitrary constants + their PI
			$x = 0, y = 0$: $C = \frac{1}{8}$	M1	Use condition
			$\dot{x} = e^{-4t}(-3C\sin 3t + 3D\cos 3t) -$ $4e^{-4t}(C\cos 3t + D\sin 3t) + \frac{5}{8}\sin 5t$	M1	Differentiate using product rule
			$\dot{x} = 0.25, t = 0$: $0.25 = 3D - 4C$ $(D = \frac{1}{4})$	M1	Use condition (must use 0.25)
			$x = e^{-4t}\left(\frac{1}{8}\cos 3t + \frac{1}{4}\sin 3t\right) - \frac{1}{8}\cos 5t$	A1	cao
				[10]	
	(v)		Oscillations of approximately constant amplitude $\frac{1}{8}$	B1	FT their amplitude
				[1]	

Question		Answer	Marks	Guidance
2	(i)	$y' - \frac{n}{x}y = \frac{2x-1}{x} = 2 - \frac{1}{x}$	B1	Divide through by x
		IF = $e^{\int -\frac{n}{x}\mathrm{d}x}$	B1	
		$(= e^{-n\ln x}) = x^{-n}$	B1	
		$\frac{\mathrm{d}}{\mathrm{d}x}\left(yx^{-n}\right) = 2x^{-n} - x^{-n-1}$	M1	Multiply both sides by their IF
		$yx^{-n} = \frac{2x^{1-n}}{1-n} + \frac{x^{-n}}{n} + A$	M1 A1	Integrate both sides Must include arbitrary constant
			M1	Divide both sides, including an arbitrary constant, by their IF
		$y = \frac{2x}{1-n} + \frac{1}{n} + Ax^{n}$	A1	cao
			[8]	
	(ii)	$y = 0, x = 2, n = -1: \quad (A = -2)$	M1	Use condition to find a value for A
		$y = x - 1 - \frac{2}{x}$	A1	
		Curve in 1st and 4th quadrants through (2,0)	B1	Must use correct form of solution. Sketch going through (2, 0) with positive gradient at (2, 0)
		Correct behaviour as $x \to 0$ and $x \to \infty$	B1	Ignore curve for $x < 0$
			[4]	
	(iii)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(yx^{-1}\right) = 2x^{-1} - x^{-2}$	M1	Find and multiply by IF
			A1	
		$yx^{-1} = 2\ln x + x^{-1} + B$	M1	Integrate both sides
			A1	Must include arbitrary constant
		$y = 2x\ln x + 1 + Bx$	A1	cao
			[5]	

Question		Answer	Marks	Guidance
	(iv)	$B=-1$	M1	Use condition $x=1, y=0$
		$y'=2\ln x+2-1=0$	M1	Differentiate, equate to zero
		$\ln x=-\frac{1}{2}$: One solution	E1	
			[3]	
	(v)	y' values: 1	M1	NB the DE is used in the form $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}\left(y+\frac{1}{\sqrt{2x-1}}\right)$
		0.92079 or 0.192079	A1	Agree to 3 s.f.
		0.86436	A1	Agree to 3 s.f.
		y value: 0.2785	A1	0.279 (or better)
			[4]	
3	(i)	$20v\frac{\mathrm{d}v}{\mathrm{d}x}=100-4v^2$	M1*	Use N2L with accn in terms of v and x
		$\frac{5v\mathrm{d}v}{\left(25-v^2\right)}=\mathrm{d}x$	M1dep *	Separate variables
		$-\frac{5}{2}\ln\left(25-v^2\right)=x+A$	M1dep *	Integrate both sides
			A1	lhs
			A1	rhs, including $+A$
		$A=-\frac{5}{2}\ln 25$	M1dep *	Use condition
		$25-v^2=25\mathrm{e}^{-\frac{2}{5}x}$	M1dep *	rearrange
		$v^2=25\left(1-\mathrm{e}^{-\frac{2}{5}x}\right)$	E1	cao
		When x = 10, $v=4.95(4)$	B1	
			[9]	

Question		Answer	Marks	Guidance
	(ii)	$20\dfrac{dv}{dt}=100-4v^2$	M1*	Use N2L with accn in terms of v and t
		$\dfrac{5dv}{(25-v^2)}=dt$	M1dep*	Separate variables
		$\dfrac{1}{2}\ln\left(\dfrac{5+v}{5-v}\right)=t+B$	M1dep*	Integrate both sides. The integral may be quoted from MF2 or PF $\dfrac{1}{2}\left(\dfrac{1}{5-v}+\dfrac{1}{5+v}\right)$ used.
			A1	Correct expression
		$B = 0$	M1dep*	Use condition
		$t=\dfrac{1}{2}\ln\left(\dfrac{5+v}{5-v}\right)$	A1	aef
		When v =4.954, t = 2.689	M1 **[7]**	Use answer from **(i)**
	(iii)	$20\dfrac{dv}{dt}=100-2v$	M1	Use N2L with accn in terms of v and t
		$-10\ln(50-v)=t+C$	M1	Separate and integrate
		$C=-10\ln 50$	M1	Use condition
		$t=10\ln\left(\dfrac{50}{50-v}\right)$	A1	
		$v=50\left(1-e^{-\frac{1}{10}t}\right)$	M1	Make v subject
			A1	Correct expression
		Terminal velocity = 50ms^{-1}	B1 **[7]**	

Question			Answer	Marks	Guidance	
			OR: $20\dfrac{\mathrm{d}v}{\mathrm{d}t}=100-2v$	M1	Use N2L with accn in terms of v and t	
			[IF: $\mathrm{e}^{-0.1t}$: $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(v\mathrm{e}^{-0.1t}\right)=5\mathrm{e}^{-0.1t}$]			
			$v\mathrm{e}^{-0.1t}=-50\mathrm{e}^{-0.1t}+A$	M1 A1	Multiply through by IF and integrate	
			$A=50$	M1	Use condition	
			$v=50\left(1-\mathrm{e}^{-\frac{1}{10}t}\right)$	M1	Make v subject	
				A1	Correct expression	
			Terminal velocity = $50\mathrm{ms}^{-1}$	B1		
				[7]		
			OR: $20\dfrac{\mathrm{d}v}{\mathrm{d}t}=100-2v$	M1	Use N2L with accn in terms of v and t	
			Auxiliary eqn $10m+1=0$: CF: $v=A\mathrm{e}^{-0.1t}$	M1		
			PI: $v=B$: $B=50$	M1		
			GS: $v=50+A\mathrm{e}^{-0.1t}$	A1		
			$A=-50$	M1	Use condition	
			$v=50\left(1-\mathrm{e}^{-\frac{1}{10}t}\right)$	A1	Correct expression	
			Terminal velocity = $50\mathrm{ms}^{-1}$	B1		
				[7]		
	(iv)		When v = 4.954, $t=10\ln 1.11=1.04\,\mathrm{s}$	B1 **[1]**		

Question			Answer	Marks	Guidance
4	(i)		$\frac{d^2x}{dt^2}=2\frac{dx}{dt}-2\frac{dy}{dt}$	M1	Differentiate
			$=2\frac{dx}{dt}-\frac{3}{4}x+60e^{-\frac{1}{2}t}$	M1	Substitute for $\frac{dy}{dt}$
			$\frac{d^2x}{dt^2}-2\frac{dx}{dt}+\frac{3}{4}x=60e^{-\frac{1}{2}t}$	A1	AG Rearrange
			Auxiliary equation $m^2-2m+\frac{3}{4}=0$	M1	
			$m=\frac{1}{2},\frac{3}{2}$	A1	
			CF: $x=Ae^{\frac{3}{2}t}+Be^{\frac{1}{2}t}$	F1	
			PI: $x=Pe^{-\frac{1}{2}t}$	B1	Correct form for their CF
			$x'=-\frac{1}{2}Pe^{-\frac{1}{2}t}, x''=\frac{1}{4}Pe^{-\frac{1}{2}t}$	M1	Differentiate and substitute
			$P=30$	A1	Solve
			$x=Ae^{\frac{3}{2}t}+Be^{\frac{1}{2}t}+30e^{-\frac{1}{2}t}$	F1	PI + CF with 2 arb const
				[10]	
	(ii)		$y=x-\frac{1}{2}\frac{dx}{dt}$	M1	Rearrange
			Substitute for x and $\frac{dx}{dt}$	M1	
			$y=\frac{1}{4}Ae^{\frac{3}{2}t}+\frac{3}{4}Be^{\frac{1}{2}t}+\frac{75}{2}e^{-\frac{1}{2}t}$	A1	cao. As final answer
				[3]	

Question			Answer	Marks	Guidance	
	(iii)		$x=40, t=0 \Rightarrow 40=A+B+30$	M1	Use condition	
			$y=50, t=0 \Rightarrow 50=\frac{1}{4}A+\frac{3}{4}B+\frac{75}{2}$	M1	Use condition	
			$A=-10, B=20$			
			$x=-10e^{\frac{3}{2}t}+20e^{\frac{1}{2}t}+30e^{-\frac{1}{2}t}$	A1	cao	
			$y=-\frac{5}{2}e^{\frac{3}{2}t}+15e^{\frac{1}{2}t}+\frac{75}{2}e^{-\frac{1}{2}t}$	A1	cao	
				[4]		
	(iv)		When $x=0$ $20e^{T}-10e^{2T}+30=0$	M1	Multiply by $e^{\frac{1}{2}T}$	
			$e^{2T}-2e^{T}-3=0$	M1	Attempt to solve as a quadratic	
			$e^{T}=3$ (or -1)	A1		
			$T=\ln 3$ (= 1.10)	A1	cao	
				M1	Substitute *T* in expression for *y*	
			$y=34.64$	A1	cao	
				[6]		
	(v)		Unsuitable, X is negative	B1		
				[1]		

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4758 Differential Equations (Written Examination)

General Comments

The standard of the responses on this paper were of a pleasingly high standard, and many candidates scored full marks on some or all of the questions. The methods required to solve the second order differential equations in Questions 1 and 4 were known by almost all candidates and these two questions were attempted by the majority of the candidates. Questions 2 and 3 were chosen by candidates in roughly equal measure. Each of these two questions had parts that some candidates found quite challenging.

Comments on Individual Questions

Question 1

Second order linear differential equations

(i) All candidates were familiar with the method of solution required in this part. Any marks lost were because of arithmetical errors in solving either the equations to find the particular integral or the quadratic equation to find the roots of the auxiliary equation.

(ii) The maximum displacement of the particle was most easily found by differentiating the solution, $x = \frac{1}{12}e^{-4t}\sin 3t$, found in part (i) and equating this derivative to zero. This resulted in the simple trigonometric equation, $\tan 3t = 0.75$, with solution $t = 0.2145$, giving x =0.0212. A significant minority of candidates simply ignored the exponential part of the solution and stated that the maximum value of x occurs when $\sin 3t = 1$.

(iii) For large values of t, the solution for x indicates oscillatory motion with decreasing amplitude. The majority of candidates stated only that x tended to zero.

(iv) As in part (i) all candidates were confident in applying the correct method of solution and any loss of marks was due to arithmetical or algebraic slips.

(v) For large values of t, this new solution for x indicates oscillatory motion with approximately constant amplitude $\frac{1}{8}$. Many candidates described this situation fully. Some candidates stated the form of the solution for large values of t but did not go on to comment on what type of motion this represented.

Question 2

First order differential equations

(i) Almost all candidates recognised that the given differential equation required the application of the integrating factor method and most began correctly by dividing through by x, the coefficient of $\frac{dy}{dx}$. Most candidates found the correct integrating factor. Some candidates made a sign error and obtained x^n instead of x^{-n}. Often it was not clear whether this was a slip or a misunderstanding of the fact that the calculation of the integrating factor must include the sign in the coefficient of x in the differential equation. A variety of errors appeared in the ensuing integration, with the negative algebraic powers of x causing more problems than might have been expected.

(ii) Most candidates applied the initial condition to their solution in part (i) and were awarded a method mark. For the sketch, follow through marks were available for any solution of the correct form. One mark was awarded for a curve that passed through the given point (2,0) with a positive gradient at that point. The second mark was awarded for a curve that had the correct asymptotic behaviour for small and large positive values of *x*. Those candidates who had obtained the correct solution in part (i) usually produced good sketches and scored full marks in this part.

(iii) Candidates were asked to consider the case $n = 1$. Some candidates applied this value of n to the first part of their work in part (i) while others began again with the new differential equation. It was interesting to note that finding the integrating factor from scratch, for this particular case, was usually done correctly, without the sign errors of part (i).

(iv) Most candidates used the initial condition and then differentiated and equated their derivative to zero. Those who worked accurately usually found the correct *x* value of the single stationary point.

(v) Almost all candidates rearranged the given differential equation into the form required to apply Euler's method and many scored full marks. Other candidates gave a list of numbers, none of which related to the correct ones, and it was not possible to award any marks. Sight of either 0.921 or 0.192(1) or equivalent was required as evidence that the method was being applied correctly.

Question 3

First order differential equations

Each of the first three parts of this question required a statement of Newton's second law of motion with the acceleration written in the appropriate form for the request, for example in part (i) involving *v* and *x*. Candidates who appreciated this usually made good progress and scored the majority of the marks.

(i) This part required a straightforward application of the method of separation of variables resulting in a logarithmic expression involving *v*. The majority of candidates who started from the correct form of the acceleration, $v\frac{\mathrm{d}v}{\mathrm{d}x}$, almost always worked accurately and found the given expression for v^2 in terms of *x*. The minority of candidates who worked with $\frac{\mathrm{d}v}{\mathrm{d}t}$ did not gain any credit in this part. However, a number of candidates did realise when they moved on to part (ii) that they had already done the work in part (i) and indicated this. An indication such as this earned the relevant marks in part (ii), but could not be credited without the candidate making it clear to the examiner that the link had been made.

(ii) This part attracted either full marks or no marks, depending on whether or not a candidate realised that the way forward was to find *v* in terms of *t* and then use the numerical value for *v* from part (i). A significant number of candidates made valiant attempts to integrate the expression for *v* in terms of *x* obtained by taking the square root of the given expression in part (i). Without exception they were unsuccessful.

(iii) Candidates were back on firmer ground in this part and most produced accurate solutions. The differential equation resulting from Newton's law can be solved by using the integrating factor; by separating the variable; or by finding the complementary function and particular integral. Most candidates opted for separating the variables, but it is worth noting that the last of the three approaches mentioned was the most straightforward. Candidates are confident in applying this method to a second order linear differential equation, but do not seem to consider it as an option when dealing with a first order linear differential equation.

(iv) This was a simple numerical substitution. Follow through was not given, so the one mark available was a reward for accurate working in parts **(i)** and **(iii).**

Question 4

Simultaneous second order linear differential equations

(i) There were many excellent responses to this part and the majority of candidates scored full marks. The most common error was a numerical slip when finding the coefficients in the particular integral.

(ii) Almost all candidates gained the two method marks and the majority also gained the accuracy mark.

(iii) All candidates made a good attempt at this part and most produced accurate solutions.

(iv) Most candidates realised that they needed to equate their expression for *x* to zero. The resulting equation involved three different exponential terms and the key to making progress was to multiply through by $e^{0.5t}$, yielding a quadratic equation in e^{t}. A pleasingly high number of candidates took this step and went on to score full marks.

(v) Many candidates made the correct comment that the number of species *X* becomes negative for times greater than *T*, indicating that they understood the implications of the predictions of the model situation that was being modelled.

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GCE Mathematics (MEI)											
				Max Mark	a	b	c	d	e	u
4751	01	C1 – MEI Introduction to advanced mathematics (AS)	Raw	72	63	58	53	48	43	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – MEI Concepts for advanced mathematics (AS)	Raw	72	56	50	44	39	34	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper	Raw	72	56	51	46	41	36	0
4753	02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – MEI Applications of advanced mathematics (A2)	Raw	90	74	67	60	54	48	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – MEI Further concepts for advanced mathematics (AS)	Raw	72	62	57	53	49	45	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – MEI Further methods for advanced mathematics (A2)	Raw	72	65	58	52	46	40	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – MEI Further applications of advanced mathematics (A2)	Raw	72	59	52	46	40	34	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) MEI Differential Equations with Coursework: Written Paper	Raw	72	63	57	51	45	38	0
4758	02	(DE) MEI Differential Equations with Coursework: Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – MEI Mechanics 1 (AS)	Raw	72	62	54	46	39	32	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – MEI Mechanics 2 (A2)	Raw	72	54	47	40	33	27	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – MEI Mechanics 3 (A2)	Raw	72	64	56	48	41	34	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – MEI Mechanics 4 (A2)	Raw	72	53	45	38	31	24	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – MEI Statistics 1 (AS)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – MEI Statistics 2 (A2)	Raw	72	65	60	55	50	46	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – MEI Statistics 3 (A2)	Raw	72	64	58	52	47	42	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – MEI Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – MEI Decision mathematics 1 (AS)	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – MEI Decision mathematics 2 (A2)	Raw	72	54	49	44	39	34	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – MEI Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) MEI Numerical Methods with Coursework: Written Paper	Raw	72	56	50	45	40	34	0
4776	02	(NM) MEI Numerical Methods with Coursework: Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) MEI Numerical Methods with Coursework: Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – MEI Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

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GCE Statistics (MEI)

				Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI (Z1)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI (Z2)	Raw	72	55	48	41	34	27	0
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI (Z3)	Raw	72	56	48	41	34	27	0
			UMS	100	80	70	60	50	40	0

GCE Quantitative Methods (MEI)

				Max Mark	a	b	c	d	e	u
G244	01	Introduction to Quantitative Methods MEI	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods MEI	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1 MEI	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision 1 MEI	Raw	72	56	51	46	41	37	0
			UMS	100	80	70	60	50	40	0